Enrollment No. ____________

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous)

Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION APRIL – 2017

M. Sc. Mathematics

16PMTCC06 – ALGEBRA -II

Duration of Exam – 3 hrs | ***Semester – II*** | ***Max. Marks – 70***

Part A (5x2= 10 marks)

Answer **ALL** questions

1. Define: (1) Division ring. (2) Characteristic of a field.
2. Find $[\mathbb{Q}(\sqrt{2}, \sqrt{3}) : \mathbb{Q}]$.
3. Define: Derivative of a polynomial.
4. Find (1) $[GF(512): GF(8)]$ (2) $[GF(729): GF(9)]$.
5. Define: Module over a ring R.

Part B (5X5= 25 marks)

Answer **ALL** questions

6a. If F is a field of characteristic p then prove that $(\alpha+\beta)^{p^n} = \alpha^{p^n} + \beta^{p^n}$, for any $\alpha, \beta \in F$.

OR

6b. If a is algebraic over a field F and $p(x) \in F[x]$ is a minimal polynomial such that $p(a) = 0$ then prove that $p(x)$ is irreducible over F.

7a. If K is a finite extension field of F then prove that K is an algebraic extension of F.

OR

7b. Define: Perfect field. Prove that any finite field is perfect.

8a. If F is a field of characteristic p and $f(x) \in F[x]$ is a polynomial such that $f'(x) = 0$ then prove that $f(x) = g(x^p)$ for some polynomial $g(x) \in F[x]$.

OR

8b. For any two polynomials $f(x)$ and $g(x)$ over a field F and for any $\alpha \in F$, prove that (1) $\big(f(x) + g(x)\big)' = f'(x) + g'(x)$ (2) $\big(\alpha f(x)\big)' = \alpha f'(x)$.

9a Prove that $\mathbb{Q}$ is a $\mathbb{Z}$-module.

OR

9b Let M be an R-module. $N \subset M$ is a submodule of M if and only if for $r_1, r_2, \dots, r_k \in R$ and $n_1, n_2, \dots, n_k \in N$, $\sum_{i=1}^{k} r_i n_i \in N$.

10a Let R be a ring with unity and M be an R-module. Prove that M is simple if and only if M is generated by any non-zero element $x \in M$.

OR

10b State and prove Schur's lemma.

Part C ***(5X7= 35 marks)***

Answer **ALL** questions

11a. Prove that $\mathbb{Z}_3(i)$ is a field.

OR

11b. If L is a finite extension of K and K is a finite extension of F then prove that L is finite
extension of F. Also prove that $[L : F] = [L : K] \cdot [K : F]$.

12a. Let K be an algebraic extension of F. If $a \in K$ is algebraic of degree n over F then prove that $[F(a): F] = n$.

OR

12b. Find splitting field of (1) $x^5 - 1$ (2) $x^4 + x^2 + 1$ over $\mathbb{Q}$.

13a. Define: (1) Algebraic extension (2) Normal extension. Prove that every algebraic extension field (of a field F) of degree 2 is normal extension.

OR

13b. Prove that a polynomial $f(x)$ over a field F has a multiple root if and only if $f(x)$ and $f'(x)$ have a nontrivial common factor in $F[x]$.

14a. Prove that $GF(p^n)$ has a unique subfield of order p^m if and only if $m|n$.

OR

14b. Define: R-homomorphism between two modules. Let M and N be two R-modules and $f: M \to N$ be an R-homomorphism. Prove that Ker f and $R(f)$ are submodules of M and N respectively.

15a. Let M and N be two R-modules and $f: M \to N$ be an R-homomorphism. Prove that
(i) $f(0) = 0$.
(ii) $f(-x) = -f(x)$.
(iii) $f(x - y) = f(x) - f(y), \forall\, x, y \in M$.

OR

15b. An R-module M is a direct sum of submodules $M_1, M_2, \ldots, M_n$ if and only if

(1) $M = M_1 + M_2 + \cdots + M_n$

(2) $M_i \cap (M_1 + M_2 + \cdots M_{i-1} + M_{i+1} + \cdots + M_n) = \{0\}, \forall i, 1 \leq i \leq n.$
